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## MATHEMATICS AND PSYCHOLOGY.

BY C. C. GROVE.

(Continued from page 182, Vol. VIII.)

### PART II.

#### THE MATHEMATICS OF PSYCHOLOGISTS AND THE VALIDITY OF THEIR USES OF ITS FORMS AND PROCESSES.

Above we pointed out in rather homiletic form an attitude toward the attacks upon the disciplinary value of mathematics and indicated the proper reaction of teachers of mathematics to those attacks. Herein we proceed to sketch the attempts at "mental measurement," to ascertain what was and is being measured, to investigate the fundamental assumptions made in the development of the statistical methods employed in the investigations on the transfer of mathematical training and to what classes of data we may apply methods involving the theory of probability. It is intended to give examples of classes of errors and misconceptions found in the literature, and it is hoped to be possible to make one or two helpful, constructive suggestions.

*The Essentials of Mental Measurement*, by WILLIAM BROWN, Cambridge University Press, 1911, gives an excellent, brief review of the work towards "mental measurement" up to 1911, but it is not easy reading for the layman on account of its brevity. A comprehensive historical and critical treatment may be found in *Experimental Psychology*, Vol. II, Part II, *Instructors' Manual*, by E. B. TITCHENER, The Macmillan Co., 1905. Only a scholarly layman in the field under discussion would get through any considerable portion of this book, however. A book that will bring the subject up to date, to appear during June, 1916, is *The Measurement of Intelligence*, by Lewis M. German, Houghton Mifflin Co.

You will therefore pardon the writer if he takes more time than he desires to take in simply sketching the history of the

attempts at "mental measurement." We shall consider, first, attempts at *direct measurement*; second, attempts at *indirect measurement*.

#### DIRECT MEASUREMENT.

The employment of advanced mathematics by psychologists may well be considered to date from Johann Friedrich Herbart (1776-1841), who carrying out his conception of a statics and a mechanics of mind as well as of matter wrote at Königsberg in 1822 a memoir with the lengthy title: *De Attentionis mensura causisque primariis. Psychologiae principia statica et mechanica exemplo illustraturus scripsit Joannes Fridericus Herbart, Philos. et Paedog. P. P. D. in Academia Regiomontana.*

It has been my privilege to examine possibly the only copy of this book in America, owned by Professor David Eugene Smith, who suggested and inspired the study of the present subject.

His employment of differential and integral equations to such a subject as attention was at once a daring and startling adventure. We may well imagine that this successor to Kant at Königsberg attracted much attention in university circles. Practically no one, however, has adopted his doctrine, and the memoir on attention within fifty years was entombed in De Morgan's "Budget of Paradoxes," with the statement that the thesis had not been upheld and with the expression of hope that De Morgan's mathematical readers might have better luck than he had had in interpreting the formulas if there is anything in them.

Like many another great pioneer, Herbart, could he return after seventy-five years, would likely say, "My psychology has indeed set men to work. They have recast much and cast some aside, but my work has not lost its influence." Men were awakened to the possibility of applying the forms of mathematics to psychology, although almost none venture to adopt Herbart's mathematical foundations. As James Ward has written, "We are most of all indebted to Herbart for the enormous advance psychology has been enabled to make thanks to his fruitful treatment of it. His criticisms are worth more than his constructions." His mathematics must be classed with his constructions.

During the same decade in which Herbart's *De Attentionis* was published, Ernst Heinrich Weber (1795-1878), professor

of anatomy at Leipzig, was making experiments in an attempt to give quantitative expression to the relation between physical stimuli and the resulting psychical reactions. His experiments were conducted chiefly with weights that were tested by the pressure they exerted and by lifting. He observed the differences in the lengths of lines that could be detected. In all these tests he found that in order that the sensational difference remain the same the change in the stimulus must be constantly in the same ratio to the preceding stimulus. For example, if weights of 32 and 30 ounces can just be distinguished, so can weights of 64 and 60 ounces, whereas a weight of 62 or 63 ounces could not be distinguished from one of 60 or 64 ounces. In the case of lengths the ratio was found to be about one one-hundredth, so that lines of 50 and 50.5, or of 100 and 101 inches, could be just distinguished.

The law may be stated in the words: In order to get successively just-noticeable differences in sensation, or so that the sensations increase by a common difference, the increases in the stimulus must be in geometrical progression; *i. e.*, for sensational intensities to increase in arithmetical progression the stimulus must increase in geometrical progression; or still further in the symbols of the differential calculus.

$dR/R = a$  constant, the just-noticeable difference in sensation,  $dS$ , where  $R$  represents the value of a stimulus and  $dR$  the just-noticeable difference in stimulus.

Weber seems to have had no special absorbing interest in his findings, at least not more than in many other observations that engaged his attention. It was left for Gustav Theodor Fechner (1801-1887) to name the doctrine Weber's Law and to call Weber the Father of Psychophysics. Fechner came to the University of Leipzig in 1834 as professor of physics, just after Weber published the final report on his experiments.

Quoting from *The Essentials of Mental Measurement*, l. c., p. 4, I add:

"Fechner verified Weber's Law in many different realms of sensation intensity, and made it the basis of his own system of mental measurement. This he did by making the following three assumptions:

"1. That a sensation intensity is a measurable magnitude and may therefore be regarded as a sum of unit intensities;

"2. That just-noticeable differences of sensation intensity are equal at different parts of the stimulus scale, and may therefore conveniently serve as the unit intensities above-mentioned ;

"3. That the just-noticeable difference of sensation may be treated as a difference of two sensations, or at least that if Weber's Law applies to the former ('sensed difference') it will also apply to the latter ('difference sensation').

"On the basis of Weber's Law and these added assumptions, Fechner obtains the following formula, viz.,

$$dS = c(dR)/R,$$

which he calls the *fundamental formula for mental measurement*." Integrating this formula, making certain assumptions as to initial values, and changing to common logarithms, the formula was expressed as  $S = k \cdot \log R$ .  $S$  stands for sensation intensity and  $R$  for stimulus as before.

All the assumptions are questionable and have been vigorously attacked. As to the first assumption, we observe crimson is not  $n$  times pink nor yet the  $n$ th power of pink. Fechner himself fell back on introspection to seek ground for his second assumption. His third assumption brings us to the interpretations of Weber's Law, in that we would know in what sense the relationship between "sensed difference" and "difference sensation" is given by the Law. Shall we with Fechner say it is a psychophysical law giving the fundamental relationship between body and mind? Or, shall we with Wundt interpret it as purely psychological, expressing the law of relativity of mind? Or, with Müller, Ebbinghaus, James, and others, do we believe the law is physiological and that we must study therefore the nature of nervous action?

But throughout we are positing Weber's Law, and that too in the face of the accepted fact that experiments have shown that it holds only within a limited range. As to the value of the doctrine as a whole, it may be interesting to quote Professor James, who wrote (*Psychology*, Vol. I, p. 534), concerning Fechner's book, *Elemente der Psychophysik*, "of which . . . the proper psychological outcome is just *nothing*. The psychophysical law controversy has prompted a good many series of observations on sense-discrimination, and has made discussion of them very vigorous. It has also cleared up our ideas about

the best methods of getting average results, when particular observations vary ; and beyond this it has done nothing."

Finally, J. R. L. Delboeuf defined a unit of sense measurement that gave some hope. He, in arranging a series of grays, took for unit of measure the *contrast* between two shades however near or remote. As then he could select another between which and the lighter of the two chosen there was as much contrast as between the two chosen, he could get two in the scale of *contrast*, etc. Thus there was, he thought, something that could be measured in terms of a unit of the same kind, and said that "the sensation is measured by a unit of sensation." Observe that this is, however, a unit of *contrast* and *not* a unit of *sensation*. And *is contrast* a measurable quantity? If we arrange objects so that with respect to some quality each is sensibly equally contrasted with respect to the object next on its right and left, can we be said to be measuring that abstract quantity, Delboeuf's "degree of sensible contrast" ?

Though a member neither of the Department of Philosophy nor of that of Psychology, the writer may perhaps be permitted to express what introspection and observation have given him as experience. In *Elements of Social Psychology*, LADD AND WOODWORTH, Scribners, 1911, Part Third, p. 668, we read: "In all the discussions of the previous chapter it was implied that we were dealing with two different existences—separable at least in thought, and apparently belonging to widely divergent species of existence." And again on page 687, "It knows no decisive reason against the belief that such a non-material and real unit-being, as the mind is, should exist in other relations than those which it sustains at present to the structure of the brain. On the contrary, it discloses certain phenomena which at least suggest, and perhaps confirm, the possibility of such existence for the Mind."

The writer would state that he experiences three different existences, which at some stages of their inter-relationships are difficult of distinction, as is the case with vegetable and animal life sometimes. This experience he chooses to state briefly in the words, man is a physical, a psychical, and a spiritual, being. The physical refers to the body ; its training has been called physical education ; the psychical has to do with the mind and we give the name education to its development ; the spiritual life is

the life of faith and its realm is religion. At present we are not concerned with the latter but mention it to round out the whole of human nature.

We add that *each realm has its own standards, measures and language and these are transferable only by analogy*. Man is not in the same stage of development in these three forms of life at any time. In the last two he comes to live in that realm where time and space are no more. He still knows very little of the language of those realms. That idea, that thought, that vision of truth, what distance did it come and what length of time did it require for the journey? This light is brighter than that and physicists measure their intensities, physical properties in terms of physical units of measure, but introspection shows that the intellectual man is perceiving not *quantitatively* but *qualitatively* the sensations produced. This light hurts my eye more than that, but the sensation, the message to the brain, the consciousness of it, is not twice as great. It is a different sensation, message, consciousness. In the telephone the diaphragm is so sensitive that even the quality, the identity of the speaker's voice is recognized. The softer tones are none the less perceived than the louder, but they are perceived as softer.

Things must be discerned in their own realms. You cannot attain unto infinity by any accumulation of discrete units, however large each unit may be. Yet infinity exists between the nibs of your pen. The finite and the infinite are two different realms. Their languages are not the same and poorly transferable by analogy. You simply cannot attain unto the infinite that way.

So there are other things that cannot be done in what may seem to some a reasonable way. Yet many of the greatest things are accomplished by apparently not being concerned with those but other less important things. "Thus life succeeds in that it seems to fail." We believe that "mental tests" are not going to "measure" mentality in an individual, whatever else they may measure, much beyond what the trained judgment of the tester tells him. The physical concomitants of mental action are not readily transferable as language nor as measures to the mental functions themselves. Thus it seems inadvisable to speak of "mental measurement," for we are therein following the vulgar practice of speaking of mental and spiritual phe-

nomena in terms of the physical realm of sense and sensation. We believe the word "measurement" should be left for the physical world and that it is better to speak of "grading" or of "ranking" judgments and mental characteristics and functions. Therein we would not intend to imply that a person receiving a grade of 80 has twice as much of the quality tested as another graded 40. We would mean that at a specified time, under specific conditions, under and in the judgment of a named person conducting the test the first person did twice as much of a test given in detail as the other. That is what we mean when we grade students and we do not profess that mental abilities or intellectualities are to each other in the given ratios.

Brown, *l. c.*, p. 10, says: "The preceding account has probably sufficed to show that purely psychical measurement is a possibility. Its practical application, however, has been more detailed than extensive." A thoughtful reading of what precedes we feel convinces the reader that we cannot measure man's intellectual stature or growth with a meter bar, a liter, nor in kilograms; also that psychic phenomena though in time and space require neither time nor space in the doing. We can measure their physical concomitants, their stimuli and physical expressions, or effects, but not the psychical, in terms of the physical. *Each realm has its own standards, measures and languages and these are transferable only by analogy.*

#### INDIRECT MEASUREMENTS.

In seeking for early work along the line of indirect "mental measurement" I find that at the meeting of the Anthropological Institute of Great Britain and Ireland\* on November 10, 1885, the president, Sir Francis Galton, said the paper "by Mrs. Bryant gives the result of a first scientific attempt to test certain elementary characteristics in the disposition of school-children, and that by Mr. Jacobs endeavors to assign a numerical ratio to the intellectual ability of the Jews as compared to that of other races." The first of these papers has been read with interest and deserves mention because it seems to have been overlooked, because it antedates other papers given as first memoirs on the subject, and because it expresses such sound judgment and ability on the part of its author, which received most favor-

\* Journal, Vol. 15, 1885-6, pp. 338 sq.



able criticism at the meeting from psychologists as well as other scientists.

Dr. Bryant's paper was written at the suggestion of Francis Galton that she "devise means of testing the mental characteristics of children." She chose the plan of having them write reports on observations made without communication at the same time, believing that "a writer tells more tales about his fundamental intellectual characteristics than a talker in close contact with another mind or other minds is likely to tell; at any rate, he tells different tales." At the outset Dr. Bryant distinguishes between "the sense impression and the apprehension of it by the mind, as between the passive and active factors of perception."

Several other quotations should be given:

"The results of a *single* test may be accidental, . . . much less importance should be attached to negative than to positive results."

"The exact numerical marks cannot be considered as at all reliable in the sense of assigning precise degrees of value, and on the whole I am inclined to think that verbal remarks would be more valuable."

"It goes without saying that the sources of error in such observations as these are very numerous. Accidental variation in the subject of the observation from time to time may produce quite misleading responses to the tests used. This is the least serious difficulty, however, since it can be dealt with, like all other similar difficulties, by taking the mean of several observations and noting at the same time the limits of variation as itself an important fact. More serious are the difficulties arising from the complex implication of mental quantities with one another, which makes it impossible to measure them separately as physical quantities are measured, or calculate them with any pretence to scientific accuracy."

Would that many more recent writers understood as clearly what may be done with "accidental errors" and did not tacitly assume that all the many sources of error are of the accidental type.

*(Continued in the next issue.)*

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